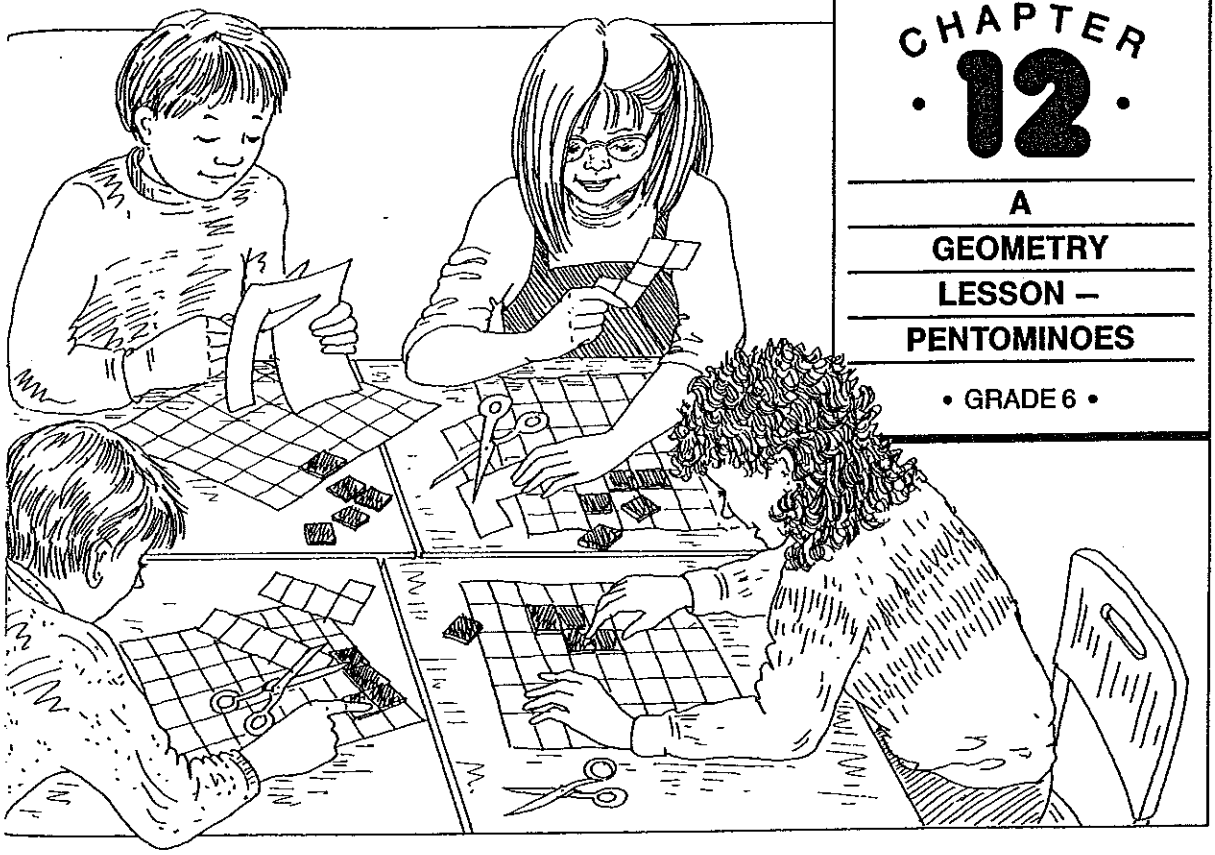


CHAPTER

12

A GEOMETRY LESSON – PENTOMINOES

• GRADE 6 •

Working with a group helps when looking for all the pentominoes.

It is important for students in the elementary grades to have hands-on experiences to develop understanding of concepts in geometry. This lesson with sixth graders models such an experience. Students use one-inch-square tiles and paper ruled into one-inch squares to solve the problem of finding all the possible ways to arrange five squares into shapes called pentominoes. This problem-solving activity gives the students a common base of experience for studying several geometric ideas.

The concept of congruence becomes key as students try to find all the different pentominoes. In order to compare the figures they create, students explore geometric transformations, including translations (slides), rotations (turns), and reflections (flips). In a follow-up activity, investigating which of their pentominoes will fold into boxes, students relate their two-dimensional exploration to three-dimensional shapes. Also, the students explore the concepts of perimeter and mirror symmetry as they sort their pentomino shapes.

This lesson is an especially rich geometry experience. It gives students the opportunity to explore several geometry concepts at the same time, which helps students to see the relationship of one concept to another. The exploration helps students develop their perception of spatial relationships. In addition, the activity necessitates students' deciding when they have found all possible pentominoes, thus requiring that they apply logical reasoning to a spatial task.

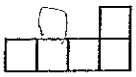
BEGINNING THE LESSON

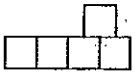
This sixth-grade class is one I have taught on a regular basis. The students are accustomed to working in small cooperative groups. Their individual desks are grouped into clusters of four, with groups of three or five as needed to accommodate all the students. The students were told yesterday that I would teach today's math lesson and that it would be a problem-solving activity to launch a unit on geometry.

When I arrived, the thirty-one students were seated in eight groups, seven groups of four and one group of three. I distributed materials to each group – twenty one-inch tiles, enough for each students to have five, and two sheets of dittoed squared paper.

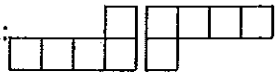
I then began the lesson by telling the class that in this problem-solving activity, they would search for different shapes that could be made from five squares, using the tiles I had distributed. "These shapes made from five squares are called pentominoes," I explained, writing the word *pentominoes* on the chalkboard. "There are three ideas you will need to understand in doing this activity.

"First of all," I continued, "there is a rule you will need to follow when making pentomino shapes. When you arrange the squares into shapes, the requirement is that at least one whole side of each square touches a whole side of another." I drew the following examples on the chalkboard and labeled them:

This is OK: 

This is not OK: 

"Second," I went on, "you will have to decide if the shapes you create are the same or different. That's where the squared paper will come in handy.

Here are two legal shapes." I drew them on the board: 

Then I cut each from a sheet of the squared paper and showed how to use the cut-out shapes to compare them.

"How many shapes are we supposed to find?" Scott asked. I, and the class, have learned to expect Scott to ask that kind of clarifying question.

"That's part of the problem for your group to solve, to find all the pentominoes and to convince yourselves that you have found all the possible arrangements there are. Let me know when you think you've done that and I'll come and discuss what your group has done."

There were no other questions.

DURING THE EXPLORATION

As the students got to work exploring the problem, the noise level rose in the room, but it was the productive kind of noise that was purposeful, not frantic. The class seemed focused and interested in the problem. With this class, as in others, whenever the students have the opportunity to handle materials, they seem to get interested more easily than on abstract, paper-and-pencil problems.

I circulated as they worked. I checked on Leah to make sure she was moving along with the others and decided that she seemed to be doing fine. During this exploring time, I stay out of their group interactions as much as possible. I listen casually to their comments, noticing how individual students are working. I note ideas I overhear that will be useful for later discussion. I am ready to offer assistance when all group members raise their hands, as they've learned to do when they need help, or when I feel a group is totally bogged down.

After almost twenty minutes, a group called me over. "We think we've found them all," they announced.

"How many do you have?" I asked.

They did a quick count of their cut-out shapes. "Nine. No, ten."

As I scanned their shapes, a nearby group that had overheard them chimed in, "We have eleven." I told the second group that I'd be there in a moment and refocused on the first group. I noticed that two of their shapes were congruent. "Examine your shapes again," I told them. "I see two that are really the same. See if you can find those, and then see if you can discover any new ones."

The group seemed a little discouraged, but Mark got them back on track. "I see them," he said, picking up the two congruent shapes. Mark is usually motivated to continue a search and can be counted on to keep a group probing. It didn't surprise me that the group with Mark in it was the first to call me over.

I moved to the second group. "We found another," they told me. "Now we have twelve, and we think we have them all." I looked over their shapes and noticed in their arrangements also that two were the same. "I see two that are congruent," I told them, moving away. I heard Susie comment, "Let's find those and then look some more. There can't be just eleven. There are never eleven of anything."

Another group signaled for me by raising their hands. It's often at this stage in a problem-solving activity that the class gets a bit hectic. This group had found eight shapes and seemed satisfied. "The group with Mark in it has found nine, and the group with Susie has found eleven," I told them. "Keep looking." They groaned a bit, but got back to work.

One more group was ready. They had found twelve, all different, and felt calmly secure and satisfied. "We think we found them all," they announced.

"Tell me why you think that," I responded.

Scott explained. "We know there is only one with all five in a row. Then we looked for shapes with four in a row. Then with three. There were lots of those. Then two. We threw out the doubles." This explanation is typically Scott. He usually provides the leadership in thinking.

"Do we have them all?" Sara asked, not having Scott's confidence and needing to know.

"Yes," I answered them, "but don't announce this to the others. Let's give them some more time to search. In the meantime here's a puzzle you can work on using your pieces. See if you can get all twelve of your pieces to fit together into one large rectangle. How many squares are there in all with your twelve pieces?"

They thought a bit. Kenny jotted down 12 x 5 on a piece of scrap paper and came up with 60. "So," I continued, "a large rectangle will use all sixty squares. What dimensions might it have?"

"Ten by six," Sara answered quickly. "Will that work?" she asked, again needing to know.

"I'm not sure," I responded honestly, "but I've fit them into a five-by-twelve rectangle and into a three by twenty. I don't know how many different ways there are to do it. Maybe you can find out."

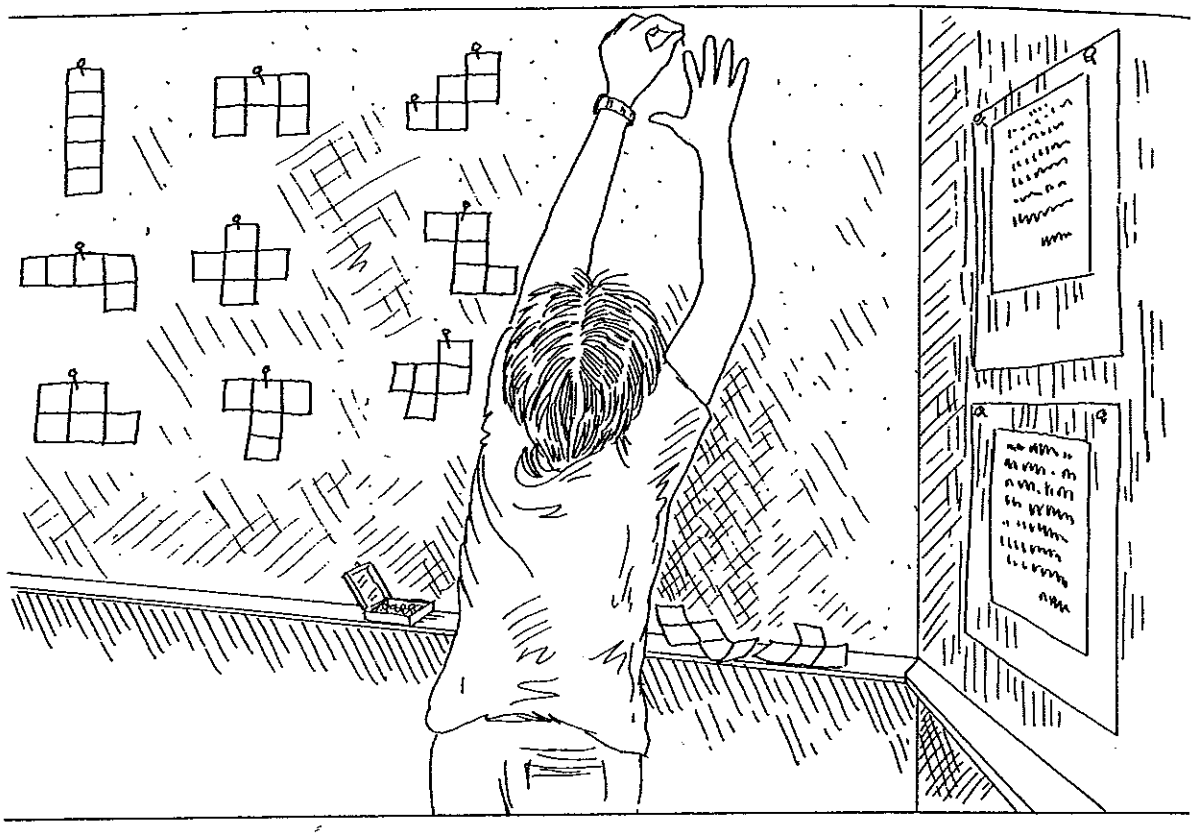
By this time two other groups had their hands raised. One was the group with Susie in it. "We have twelve now," they said. I directed them to ask the group with Scott and Sara in it to explain the puzzle activity to them. When I approached the other group, they waved me away, having gotten back to work. They had probably heard Susie's group report that they had found twelve and were willing to look some more.

One more group raised their hands to announce that they were done. They had ten shapes, all different. "I've seen two other groups with those same ten shapes, and more," I told them. "See if you can find others."

SUMMARIZING THE ACTIVITY

After another few minutes, I decided to interrupt the entire class for a discussion. It took a while to get their attention and to settle them down.

"Let's summarize this activity," I said when I had their attention. "Let's talk first about how you got organized as a group. I'd like you to hear from each other."



Mark posts his group's pentominoes for the class.

Sara started. "Scott organized us. He wanted us to look for shapes in a system. We did it his way."

"Did you understand his system?" I asked.

"Not really at first," Sara answered, "but then as we got into it, I kind of caught on. I was able to find some shapes, though."

"How about another group?" I asked.

Jennifer responded. "We just all started working, and when someone found a shape, they told the rest, and we looked to see if it was a new one or not. Then they cut it out."

"We did it the same way," Mike offered, "but Stephanie did all the cutting out."

"Why did you decide to do that?" I asked.

Stephanie answered. "We really didn't decide. I just did it." The class giggled.

"Did it work well for you?" I pursued.

The group looked at each other and nodded. "I think so," Mike said. "I liked it," Stephanie said. Stephanie likes being organized and often offers to do the kinds of chores that keep things orderly.

"Are there any other ways groups organized that were different?" I continued. I waited a bit. None was offered. "What about the problem of finding the pentominoes? Was it hard, easy, enjoyable, unpleasant? How did you feel about searching for all the shapes? Do you think your group solved the problem?" No comments were immediately offered. I waited, having learned that the students need the time to think that a few moments of silence can provide.

Finally Jennifer raised her hand. "It started off being easy, but then it got hard to find more." Some murmurs of assent went through the class.

"What did you do when it got hard?" I asked.

Jennifer responded, "Oh, we just kept looking. I found that when I just kept moving the tiles around, I would find a new shape. Then we had to check all the ones we had."

Steve agreed. "It was real hard once we got ten. We knew there were more because you had said that Mark's group had eleven."

"What did you do then?" I asked.

"We went and looked at his group's shapes," Steve confessed. Giggles again from the class.

"How did you tell which ones you didn't have?" I asked.

"That wasn't easy," Steve said. "It was hard to figure that out. We had to bring our pieces over."

"Where was I when all this was going on?" I asked, wondering how I had missed it. The class laughed.

Kirsten, also in Steve's group, offered an explanation. "Oh, you were talking to Scott's group. You were real busy."

"Are there really twelve shapes?" Leah asked.

"Yes," I replied. "How about one group that has found them all posting theirs so the other groups can see which ones they're missing. I want each group to have a complete set for tomorrow's activity. Also, I will give each of you a piece of centimeter squared paper on which you should sketch all twelve shapes for an individual record. You'll need that for tomorrow as well. There are lots of other geometry ideas I want you to investigate using your pentomino pieces. Who will post the pieces?"

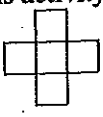
Mark volunteered to do so, and I gave him a box of pins so he could pin the pentominoes in a corner of the bulletin board. I put out a stack of centimeter paper for the students to use to make their individual recordings and suggested that one person from each group come get enough for the entire group. I had Stephanie pass out a small envelope to each group for the pentomino pieces. "Take some time now to make your own recordings and to clean up the scraps of paper," I instructed. "Label the envelope for your pentomino pieces with your group name. I'll keep the envelopes for tomorrow. If you have time and are interested in a puzzle, check with Kenny's group to learn one you can do with your pentomino pieces."

The class got busy again, finishing up, organizing. I felt good about this lesson. The students were working well together and were approaching

these kinds of problem situations more eagerly than they had when the year began. That they were able to suspend their need for the answer of how many pentominoes there were was a sign, I believed, that good attitudes toward solving problems were being developed in the class.

EXTENDING THE EXPERIENCE

As the students were finishing up, I discussed with the children's teacher how I would like to structure additional lessons. I wanted the groups to sort their shapes into two sets, those that would and those that would not fold into boxes. That way the students would experience relating two-dimensional shapes to a three-dimensional one, a cube with one face missing.

I planned to introduce this activity by starting with the shape that looks like the Red Cross symbol:  . I would ask the students to visualize how they could fold up the sides of this shape so that it would be a box without a lid. Students then would predict which side they thought would be the bottom of the box, opposite the open side, and would mark their prediction with an X on the appropriate square on their group's pentomino. Then I would model for them how to fold the pentomino shape to check their prediction.

For each of the other pentominoes, students would individually inspect the pentominoes on their individual record sheets to predict whether or not each would fold into a box. They then would compare predictions with their group members. They would discuss which squares should be marked with an X to indicate the bottom of the boxes. Finally, they would test their predictions, using their group set of cut-out pentomino shapes. Some students are very weak on these types of activities, and this sort of experience can help them strengthen those skills and build their confidence in doing so.

I had other follow-up experiences for the pentomino pieces that I planned to have the class try. The groups could sort the shapes in several ways. Students could fold each of the pieces to decide which have mirror symmetry and which do not and also find which have more than one line of symmetry. They could also sort them by their perimeters, providing a concrete experience with the notion that shapes with the same area do not necessarily have the same perimeter.

As another follow-up, each student makes a set of pentomino pieces from sturdy paper. Everyone also makes a five-by-twelve squared sheet as a gameboard, with squares matching the size of the squares used for the pentominoes. As an individual puzzle, each student uses the board and pieces, trying to fit all twelve pieces onto the board, the puzzle some groups had already begun to explore. As a two-person game, players take

turns placing pieces on the board, with the object being to play the last possible piece so that it is impossible for the opponent to fit in another. These games could go home as family gifts to help students communicate what sorts of things they were doing in their math class.

School milk cartons are useful for a further follow-up activity, for which students need to save enough school milk cartons so each student can have several. The cartons should be rinsed well, and the tops must be cut off, making them topless boxes. For this exploration students try cutting the milk cartons so they will lie flat in the different pentomino shapes they have chosen.

As long as the interest holds, pentomino pieces can be used to provide experiences with different geometric concepts. This kind of investigation, which extends over time, is valuable for giving students time to digest and process what they are learning.