Frac Jack

A fraction comparison game

(A variation by Karl Schaffer)

There are a variety of versions of Frac Jack (also spelled Frack Jack) online. This version is a bit more challenging than a number of these.

This is a game for 2, 3 , or 4 players, using a standard card deck.
The deck is shuffled and players each receive 6 cards.

Players play two cards face down at the same time; when all have placed their cards they are all turned over simultaneously. The two cards are interpreted as a fraction, the smaller number card being the numerator and the larger the denominator. The goal is to play the largest fraction and win the hand.

Aces count as 1, tens and jacks count as 10, and queens and kings count as 12. Since 12 shares factors larger than one with all but four numbers less than 12, fractions with 12 as denominator may often be reduced, which facilitates comparisons with other fractions. If a player plays two cards with the same value, that counts not as 1, but as 0, and is a losing hand. This prevents large numbers of 1s being played!

The winner of the hand takes the cards played in that hand and places them in a win-pile, not to be played any more in that game. In case of a tie, the cards are split among the winners. (If there are 4 players and 3 winners, then everyone gets two cards and the remaining two are mixed back in the deck.) At the end of the game the player with the most cards won wins the game.

Following each round each player takes two more cards from the shuffled deck.

The mathematical goal is to practice fraction comparisons, if possible without using paper and pencil, technology, or finding a least common denominator. Players must be able to explain to the other players the reason the winning fraction in a round is the largest.

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STRATEGIES

One of the goals of the game is to encourage students to apply - or even invent - comparison methods that are alternative to traditional methods, such as conversion to common denominators or conversion to decimals (which is also a form of common denominator).

Here are some sample methods which students might be encouraged to learn and use in this card game. Again, it is possible to play this game successfully without using technology, always converting to common denominators, or converting fractions to decimals in order to facilitate comparisons.

1. Common numerator. Students will most likely be familiar with comparisons of fractions which have the same denominator, in which case the one with the larger numerator is the larger fraction. However, it is also easy to compare fractions with the same numerator. For example, $\frac{3}{7}$ is greater than $\frac{3}{8}$, because $\frac{1}{7}$ is greater than $\frac{1}{8}$. (Students may like to be reminded that because $\frac{1}{7}$ of a pizza is more than $\frac{1}{8}$ of the pizza!) If numerators are equal, then the fraction with the smaller denominator is the larger number.

2. Distance from one. The common numerator method may be used to compare two fractions that are close to one. For example, $\frac{6}{7}$ is less than $\frac{7}{8}$ because $\frac{6}{7}$ is $\frac{1}{7}$ below 1, while $\frac{7}{8}$ is $\frac{1}{8}$ below 1, and $\frac{1}{7}$ is greater than $\frac{1}{8}$. Fractions like these that are close to 1 will arise frequently in the game, as players will often play two cards that are as close to 1 as possible.

3. Conversion to common numerator. Suppose you want to decide which is larger, $\frac{2}{13}$ or $\frac{1}{7}$, without finding a common denominator or converting to decimals. Instead, convert $\frac{1}{7}$ to the equivalent fraction $\frac{2}{14}$. Because $\frac{2}{13}$ is greater than $\frac{2}{14}$, it is also greater than $\frac{1}{7}$.

4. Comparisons to $\frac{1}{2}$ and mixed number numerators. Suppose you want to decide which is larger, $\frac{4}{9}$ or $\frac{3}{7}$. We will compare each to the somewhat complicated fraction $\frac{1}{2} = \frac{4+1/2}{9} = \frac{3+1/2}{7}$. Since $\frac{4}{9}$ is $\frac{1/2}{9}$ less than $\frac{4+1/2}{9}$, while $\frac{3}{7}$ is $\frac{1/2}{7}$ less than $\frac{3+1/2}{7}$, and $\frac{1/2}{9}$ is less than $\frac{1/2}{7}$, we must have that $\frac{4}{9}$ is greater than $\frac{3}{7}$.

5. Combination of methods. Suppose we are comparing $\frac{7}{9}$ and $\frac{6}{7}$. Note that $\frac{7}{9} = 1 - \frac{2}{9}$, while $\frac{6}{7} = 1 - \frac{1}{7} = 1 - \frac{2}{14}$, and since $\frac{2}{9}$ is greater than $\frac{2}{14}$, it must be the case that $\frac{7}{9}$ is less than $\frac{6}{7}$.

6. Shortcut to common denominators. in the example above, since (6)(9) > (7)(7), it must be that $\frac{7}{9}$ is less than $\frac{6}{7}$. (Can you explain why that method works?) Note that this involves "cross-multiplication," a phrase students often memorize without understanding, so this method should be used carefully and only if students can explain why it works. (This is also a shortcut to common numerators – why?)

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